

MNS Matrix and Froggatt–Nielsen Mechanism

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Abstract

Fermion mass spectra and flavor mixings are studied in detail in the $SU(6) \times SU(2)_R$ model, in which state-mixings beyond the MSSM take place. Characteristic patterns of fermion spectra is attributed to the hierarchical structure of effective Yukawa interactions via the Froggatt–Nielsen mechanism and also to the state-mixings beyond the MSSM. It is found that the neutrino mass matrix in the mass diagonal basis for charged leptons has no hierarchical structure. This is due to the cancellation among the hierarchical factors by the seesaw mechanism with the hierarchical Majorana mass matrix of R-handed neutrinos. As a consequence, V_{MNS} exhibits large mixing. It is shown that a simple setting of parameters in the present model is consistent with all observed values of fermion masses and mixings.

1 Introduction

Understanding the characteristic structure of fermion masses and flavor mixings is one of the major outstanding problems in particle physics. Up-type quarks, down-type quarks, charged leptons and neutrinos have distinct hierarchical mass patterns from each other. Moreover, the observed flavor mixing is small for quarks but large for leptons. This situation is in sharp contrast to a naive expectation from quark–lepton unification. We can disentangle ourselves from this discordance by considering the state-mixings between quarks (leptons) and extra particles beyond the minimal supersymmetric standard model

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(MSSM). In fact, it was shown in the context of $SU(6) \times SU(2)_R$ string-inspired model, which contains massless particles beyond the MSSM, that we are able to explain characteristic patterns of the observed mass spectra and mixing matrices of quarks and leptons [1, 2, 3, 4, 5]. In the model the Froggatt–Nielsen (F–N) mechanism [6] plays an important role. In Ref.[5] it was predicted that the absolute value of (1, 3) element of the MNS matrix [7] $|U_{e3}| = |\sin \theta_{13}|$ lies between λ and λ^2 , where $\lambda = 0.23$. Recently, new data on neutrino mixings become available [8]. The observed value of mixing angle $|\sin \theta_{13}|$ turned out to be $\sim \lambda^{1.3}$, which supports this prediction.

In this paper we carry out more detailed study of fermion mass hierarchies and flavor mixings in the $SU(6) \times SU(2)_R$ string-inspired model. The hierarchical structure due to the F–N mechanism comes out not only in the effective Yukawa couplings but also in the R-handed neutrino Majorana mass matrix. In the neutrino sector, where the seesaw mechanism [9] is at work, the hierarchical factors (the F–N factors) in the Dirac mass matrix are faced with the inverse of those in the Majorana mass matrix. This situation in the neutrino sector brings about a significant feature peculiar to the MNS matrix, on which main emphasis is placed in this paper. As will be shown later, the neutrino mass matrix takes the form

$$\mathcal{M}_\nu \propto \Lambda_\kappa \Sigma \Lambda_\kappa \quad (1)$$

in the mass diagonal basis for charged leptons with

$$\Lambda_\kappa = \text{diag}(\kappa_1, \kappa_2, 1), \quad (2)$$

$$\Sigma \simeq \begin{pmatrix} 1 & 0 & 0 \\ -\sigma_1 & 1 & 0 \\ \sigma_3 & -\sigma_4 & 1 \end{pmatrix} \times N^{-1} \times \begin{pmatrix} 1 & -\sigma_1 & \sigma_3 \\ 0 & 1 & -\sigma_4 \\ 0 & 0 & 1 \end{pmatrix}, \quad (3)$$

where the parameters κ_i ($i = 1, 2$) and σ_i ($i = 1, 3, 4$) are $\mathcal{O}(1)$. The above form of \mathcal{M}_ν is derived as a consequence of the fact that the F–N factors appearing in the charged lepton masses cancel out in large part through the seesaw mechanism. The matrix N in Eq.(3), in which the F–N factors are eliminated from R-handed neutrino Majorana mass matrix, has no hierarchical structure and $\det N = 1$.

As an example of parameter choice, taking a simple case that $N = \Lambda_\kappa = \mathbf{1}$ and putting $\sigma_1 = \sigma_4 = 2.2$ and $\sigma_3 = 1.2$, we obtain a large mixing solution of the MNS matrix

$$V_{\text{MNS}} = \begin{pmatrix} 0.865 & 0.471 & 0.172 \\ -0.455 & 0.594 & 0.663 \\ 0.211 & -0.652 & 0.728 \end{pmatrix}$$

for the normal hierarchy. All elements of V_{MNS} offered here are accommodated to the present experimental data [10] within 20% provided that $\delta_{CP} = 0$. In addition, eigenvalues of Σ lead to

$$\left(\frac{\Delta m_{32}^2}{\Delta m_{21}^2}\right)^{1/2} = 5.57,$$

which also coincides with the observed value. It is also found that a simple choice of the parameter values is consistent with all observed values of fermion masses and mixings.

This paper is organized as follows. In section 2 we briefly review the $SU(6) \times SU(2)_R$ string-inspired model together with the F–N mechanism. Taking the mass matrix of up-type quarks as an example, we illustrate the whole scheme of the present model. For comparison with the case of the lepton sector, we study the mass matrix of down-type quarks in section 3. State-mixings occur between down-type quarks and colored Higgsinos with even R-parity. We give the explicit form of the CKM matrix, which proves to exhibit small mixing. Similarly to the case of down-type quarks, state-mixings take place between leptons and $SU(2)_L$ -doublet Higgsinos. The mass matrix in the charged lepton sector is studied in section 4. Section 5 deals with the neutrino sector in which state-mixings enter into the seesaw mechanism. The present approach provides a phenomenological framework which enable us to analyze many experimental data. Numerical analysis of the MNS matrix and fermion spectra is given in section 6. Section 7 is devoted to summary.

2 Model and Froggatt–Nielsen mechanism

In this study we choose $SU(6) \times SU(2)_R$ as a unification gauge symmetry at the underlying string scale M_S , which can be derived from the perturbative heterotic superstring theory via the flux breaking [1]. In terms of E_6 we set matter superfields which consist of three families and one vector-like multiplet, i.e.,

$$3 \times \mathbf{27}(\Phi_{1,2,3}) + (\mathbf{27}(\Phi_0) + \overline{\mathbf{27}}(\overline{\Phi})). \quad (4)$$

The superfields Φ are decomposed into two multiplets of $G = SU(6) \times SU(2)_R$ as

$$\Phi(\mathbf{27}) = \begin{cases} \phi(\mathbf{15}, \mathbf{1}) & : \quad \{Q, L, g, g^c, S\}, \\ \psi(\overline{\mathbf{6}}, \mathbf{2}) & : \quad \{(U^c, D^c), (N^c, E^c), (H_u, H_d)\}, \end{cases} \quad (5)$$

where g, g^c and H_u, H_d represent colored Higgs and $SU(2)_L$ -doublet Higgs superfields, respectively. Doublet Higgs and color-triplet Higgs fields belong to different representations

of G and this situation is favorable to solve the triplet-doublet splitting problem. The superfields N^c and S are R-handed neutrinos and $SO(10)$ -singlets, respectively. Although D^c and g^c as well as L and H_d have the same quantum numbers under the standard model gauge group $G_{\text{SM}} = SU(3)_c \times SU(2)_L \times U(1)_Y$, they belong to different irreducible representations of G . We assign odd R-parity to the superfields $\Phi_{1,2,3}$ and even to Φ_0 and $\bar{\Phi}$, respectively. Since ordinary Higgs doublets have even R-parity, they are contained in Φ_0 . It is assumed that R-parity remains unbroken down to the electroweak scale.

The gauge symmetry G is spontaneously broken in two steps at the scales $\langle S_0 \rangle = \langle \bar{S} \rangle$ and $\langle N_0^c \rangle = \langle \bar{N}^c \rangle$ as

$$G = SU(6) \times SU(2)_R \xrightarrow{\langle S_0 \rangle} SU(4)_{\text{PS}} \times SU(2)_L \times SU(2)_R \xrightarrow{\langle N_0^c \rangle} G_{\text{SM}}, \quad (6)$$

where $SU(4)_{\text{PS}}$ represents the Pati-Salam $SU(4)$ [11]. Hereafter it is supposed that the symmetry breaking scales are $\langle S_0 \rangle = 10^{17-18} \text{GeV}$ and $\langle N_0^c \rangle = 10^{15-17} \text{GeV}$. We have two types of gauge invariant trilinear couplings

$$\begin{aligned} (\phi(\mathbf{15}, \mathbf{1}))^3 &= QQg + Qg^c L + g^c g S, \\ \phi(\mathbf{15}, \mathbf{1})(\psi(\bar{\mathbf{6}}, \mathbf{2}))^2 &= QH_d D^c + QH_u U^c + LH_d E^c + LH_u N^c \\ &\quad + SH_u H_d + g N^c D^c + g E^c U^c + g^c U^c D^c \end{aligned} \quad (7)$$

in the superpotential W .

From the viewpoint of the string unification theory, it is probable that the hierarchical structure of Yukawa couplings is attributed to some kind of flavor symmetries at the string scale M_S . If the flavor symmetry contains Abelian groups, the F–N mechanism works for the interactions among quarks, leptons and Higgs fields [6]. Accordingly, effective Yukawa interactions arise from non-renormalizable terms which respect the flavor symmetry.

Here we consider the effective Yukawa interactions for up-type quarks given by

$$W_U = \sum_{i,j=1}^3 \mathcal{M}_{ij} Q_i U_j^c H_{u0}, \quad (8)$$

where the subscripts i and j are the generation indices [1]. Due to the F–N mechanism, the matrix \mathcal{M} takes the form

$$\mathcal{M} = f_M \Gamma_1 M \Gamma_2. \quad (9)$$

Our basic assumption is that the hierarchical structure of all 3×3 mass matrices is attributed to the F–N factors Γ_1 and/or Γ_2 . Hence, hierarchy of \mathcal{M} stems only from Γ_1 and Γ_2 , and M contains no hierarchical structure. Here we put a factor f_M in order to set $\det M = 1$. The F–N factors Γ_1 and Γ_2 are described as

$$\Gamma_1 = \text{diag}(x^{\alpha_1}, x^{\alpha_2}, 1), \quad \Gamma_2 = \text{diag}(x^{\beta_1}, x^{\beta_2}, 1), \quad (10)$$

where x is given by

$$x = \frac{\langle S_0 \rangle \langle \bar{S} \rangle}{M_S^2} < 1$$

and $(S_0 \bar{S})$ is a G -invariant with a nonzero flavor charge. The exponents $\alpha_1, \alpha_2, \beta_1, \beta_2$ in the F-N factors are determined by assigning flavor charges to the matter fields. Even if x by itself is not a very small number, physical parameters can be very small if they depend on high powers of x . We assume the hierarchical patterns

$$x^{\alpha_1} \ll x^{\alpha_2} \ll 1, \quad x^{\beta_1} \ll x^{\beta_2} \ll 1 \quad (11)$$

by suitably chosen flavor charges. In this paper we ignore the phase factors of vacuum expectation values (VEV's).

The mass matrix \mathcal{M} is diagonalized via bi-unitary transformation as

$$\mathcal{V}_u^{-1} \mathcal{M} \mathcal{U}_u = \Lambda_u, \quad v_{u0} \Lambda_u = \text{diag}(m_u, m_c, m_t) \quad (12)$$

with $v_{u0} = \langle H_{u0} \rangle$. Up-type quark masses are given by

$$(m_u, m_c, m_t) \simeq v_{u0} f_M \times \left(\frac{1}{\bar{m}_{11}} x^{\alpha_1 + \beta_1}, \frac{\bar{m}_{11}}{m_{33}} x^{\alpha_2 + \beta_2}, m_{33} \right), \quad (13)$$

where $m_{ij} = (M)_{ij}$ and $\bar{m}_{ij} = \Delta(M)_{ij}^*$ [1, 2, 3]. $\Delta(M)_{ij}$'s are the cofactors of M . Diagonalization matrix is described in terms of column vectors $\mathbf{w}_i^{(u)}$ ($i = 1, 2, 3$) as

$$\mathcal{V}_u = (\mathbf{w}_1^{(u)}, \mathbf{w}_2^{(u)}, \mathbf{w}_3^{(u)}), \quad (14)$$

where $\mathbf{w}_i^{(u)}$'s are eigenvectors of $\mathcal{M} \mathcal{M}^\dagger$ and expressed as

$$\mathbf{w}_1^{(u)} = N_1^{(u)} \begin{pmatrix} 1 \\ u_1^{(u)} \\ v_1^{(u)} \end{pmatrix}, \quad \mathbf{w}_2^{(u)} = N_2^{(u)} \begin{pmatrix} u_2^{(u)} \\ 1 \\ v_2^{(u)} \end{pmatrix}, \quad \mathbf{w}_3^{(u)} = N_3^{(u)} \begin{pmatrix} u_3^{(u)} \\ v_3^{(u)} \\ 1 \end{pmatrix}$$

with

$$\begin{aligned} u_1^{(u)} &\simeq x^{\alpha_1 - \alpha_2} \frac{\bar{m}_{21}}{\bar{m}_{11}}, & v_1^{(u)} &\simeq x^{\alpha_1} \frac{\bar{m}_{31}}{\bar{m}_{11}}, \\ u_2^{(u)} &\simeq -(u_1^{(u)})^*, & v_2^{(u)} &\simeq -x^{\alpha_2} \frac{m_{23}^*}{m_{33}^*}, \\ u_3^{(u)} &\simeq x^{\alpha_1} \frac{m_{13}}{m_{33}}, & v_3^{(u)} &\simeq -(v_2^{(u)})^* \end{aligned}$$

and

$$N_i^{(u)} = \left(1 + |u_i^{(u)}|^2 + |v_i^{(u)}|^2 \right)^{-1/2}, \quad (i = 1, 2, 3).$$

3 The CKM matrix

For comparison with the case of the lepton sector, we study the mass matrix of down-type quarks in this section. At energies below the scale $\langle N_0^c \rangle$ there appear mixings between D^c and g^c which are $SU(2)_L$ -singlets.* Effective Yukawa interactions among down-type colored fields are of the forms

$$W_D = \sum_{i,j=1}^3 [\mathcal{Z}_{ij} g_i g_j^c S_0 + \mathcal{M}_{ij} (g_i D_j^c N_0^c + Q_i D_j^c H_{d0})], \quad (15)$$

where $\mathcal{Z} = f_Z \Gamma_1 Z \Gamma_1$ and $\det Z = 1$. As explained in the previous section, there is no hierarchical structure in Z . The mass matrix of down-type colored fields is given by the 6×6 matrix [1, 2, 3]

$$\widehat{\mathcal{M}}_d = \begin{matrix} & g^c & D^c \\ \begin{matrix} g \\ D \end{matrix} & \begin{pmatrix} \rho_S \mathcal{Z} & \rho_N \mathcal{M} \\ 0 & \rho_d \mathcal{M} \end{pmatrix} \end{matrix} \quad (16)$$

in unit of M_S , where $\rho_S = \langle S_0 \rangle / M_S$, $\rho_N = \langle N_0^c \rangle / M_S$ and $\rho_d = \langle H_{d0} \rangle / M_S = v_{d0} / M_S$.

The above mass matrix $\widehat{\mathcal{M}}_d$ is diagonalized via bi-unitary transformation as

$$\widehat{\mathcal{V}}_d^{-1} \widehat{\mathcal{M}}_d \widehat{\mathcal{U}}_d = \text{diag}(\Lambda_d^{(0)}, \epsilon_d \Lambda_d^{(2)}), \quad (17)$$

where $\epsilon_d = \rho_d / \rho_N = v_{d0} / \langle N_0^c \rangle = \mathcal{O}(10^{-15})$. To solve the eigenvalue problem, it is convenient to take $\widehat{\mathcal{M}}_d \widehat{\mathcal{M}}_d^\dagger$ and express it as

$$\widehat{\mathcal{M}}_d \widehat{\mathcal{M}}_d^\dagger = \begin{pmatrix} A_d + B_d & \epsilon_d B_d \\ \epsilon_d B_d & \epsilon_d^2 B_d \end{pmatrix}, \quad (18)$$

where $A_d = |\rho_S|^2 \mathcal{Z} \mathcal{Z}^\dagger$ and $B_d = |\rho_N|^2 \mathcal{M} \mathcal{M}^\dagger$. Since ϵ_d is a very small number, we can carry out our calculation by using perturbative ϵ_d -expansion. Among six eigenvalues

*An early attempt of explaining the CKM matrix via D^c - g^c mixings has been made in Ref.[12], in which a SUSY $SO(10)$ model was taken.

three of them represent heavy modes with the GUT scale masses. The remaining three, corresponding to down-type quarks, are derived from diagonalization of the $\epsilon_d^2 (A_d^{-1} + B_d^{-1})^{-1}$, namely

$$\epsilon_d^2 \mathcal{V}_d^{-1} (A_d^{-1} + B_d^{-1})^{-1} \mathcal{V}_d = (\epsilon_d \Lambda_d^{(2)})^2. \quad (19)$$

Down-type quark masses are given by $M_S \epsilon_d \Lambda_d^{(2)} = \text{diag}(m_d, m_s, m_b)$. Explicit forms are

$$(m_d, m_s, m_b) \simeq v_{d0} f_M x^{\beta_1} \times \left(\frac{1}{\sqrt{a}} x^{\alpha_1}, \sqrt{\frac{a}{b}} x^{\alpha_2}, \sqrt{\frac{b}{c}} \right), \quad (20)$$

where

$$\begin{aligned} a &= \xi_d^2 |\bar{z}_{11}|^2 + |\bar{m}_{11}|^2, & b &= \xi_d^2 |(\bar{\mathbf{z}}_1 \times \bar{\mathbf{m}}_1)_3|^2, \\ c &= \xi_d^4 x^{2(\alpha_1 - \alpha_2)} |(\mathbf{z}_3^* \cdot \bar{\mathbf{m}}_1)|^2 + \xi_d^2 x^{2(\beta_1 - \beta_2)} |(\mathbf{m}_3^* \cdot \bar{\mathbf{z}}_1)|^2, \\ \xi_d^2 &= \left| \frac{\rho_N f_M}{\rho_S f_Z} \right|^2 x^{2(\beta_1 - \alpha_1)}. \end{aligned} \quad (21)$$

Here we use the notations $z_{ij} = (Z)_{ij}$, $\bar{z}_{ij} = \Delta(Z)_{ij}^*$ and

$$\begin{aligned} \mathbf{m}_i &= (m_{1i}, m_{2i}, m_{3i})^T, & \bar{\mathbf{m}}_i &= (\bar{m}_{1i}, \bar{m}_{2i}, \bar{m}_{3i})^T, \\ \mathbf{z}_i &= (z_{1i}, z_{2i}, z_{3i})^T, & \bar{\mathbf{z}}_i &= (\bar{z}_{1i}, \bar{z}_{2i}, \bar{z}_{3i})^T. \end{aligned}$$

Diagonalization matrix \mathcal{V}_d is of the form

$$\mathcal{V}_d = (\mathbf{w}_1^{(d)}, \mathbf{w}_2^{(d)}, \mathbf{w}_3^{(d)}), \quad (22)$$

$$\mathbf{w}_1^{(d)} = N_1^{(d)} \begin{pmatrix} 1 \\ u_1^{(d)} \\ v_1^{(d)} \end{pmatrix}, \quad \mathbf{w}_2^{(d)} = N_2^{(d)} \begin{pmatrix} u_2^{(d)} \\ 1 \\ v_2^{(d)} \end{pmatrix}, \quad \mathbf{w}_3^{(d)} = N_3^{(d)} \begin{pmatrix} u_3^{(d)} \\ v_3^{(d)} \\ 1 \end{pmatrix},$$

where

$$\begin{aligned} u_1^{(d)} &\simeq x^{\alpha_1 - \alpha_2} \frac{\xi_d^2 \bar{z}_{21} \bar{z}_{11}^* + \bar{m}_{21} \bar{m}_{11}^*}{\xi_d^2 |\bar{z}_{11}|^2 + |\bar{m}_{11}|^2}, & v_1^{(d)} &\simeq x^{\alpha_1} \frac{\xi_d^2 \bar{z}_{31} \bar{z}_{11}^* + \bar{m}_{31} \bar{m}_{11}^*}{\xi_d^2 |\bar{z}_{11}|^2 + |\bar{m}_{11}|^2}, \\ u_2^{(d)} &\simeq -(u_1^{(d)})^*, & v_2^{(d)} &\simeq -x^{\alpha_2} \frac{(\bar{\mathbf{z}}_1 \times \bar{\mathbf{m}}_1)_2}{(\bar{\mathbf{z}}_1 \times \bar{\mathbf{m}}_1)_3}, \\ u_3^{(d)} &\simeq x^{\alpha_1} \frac{(\bar{\mathbf{z}}_1 \times \bar{\mathbf{m}}_1)_1^*}{(\bar{\mathbf{z}}_1 \times \bar{\mathbf{m}}_1)_3^*}, & v_3^{(d)} &\simeq -(v_2^{(d)})^* \end{aligned}$$

and

$$N_i^{(d)} = \left(1 + \left|u_i^{(d)}\right|^2 + \left|v_i^{(d)}\right|^2\right)^{-1/2}, \quad (i = 1, 2, 3).$$

We are now in a position to calculate the CKM mixing matrix [13] as

$$V_{\text{CKM}} = \mathcal{V}_u^{-1} \mathcal{V}_d = \mathcal{V}_u^\dagger \mathcal{V}_d. \quad (23)$$

Thus

$$(V_{\text{CKM}})_{ij} = \mathbf{w}_i^{(u)*} \cdot \mathbf{w}_j^{(d)}. \quad (24)$$

More explicitly, we have [3]

$$\begin{aligned} V_{us} = (V_{\text{CKM}})_{12} &\simeq x^{\alpha_1 - \alpha_2} \frac{\xi_d^2 \bar{z}_{11} (\bar{\mathbf{z}}_1 \times \bar{\mathbf{m}}_1)_3^*}{\bar{m}_{11}^* a}, \\ V_{cb} = (V_{\text{CKM}})_{23} &\simeq x^{\alpha_2} \frac{\bar{m}_{11}^* (\mathbf{m}_3 \cdot \bar{\mathbf{z}}_1^*)}{m_{33} (\bar{\mathbf{z}}_1 \times \bar{\mathbf{m}}_1)_3^*}, \\ V_{cd} = (V_{\text{CKM}})_{21} &\simeq -(V_{us})^*, \\ V_{ts} = (V_{\text{CKM}})_{32} &\simeq -(V_{cb})^*, \\ V_{td} = (V_{\text{CKM}})_{31} &\simeq (V_{us} V_{cb})^*, \\ V_{ub} = (V_{\text{CKM}})_{13} &\simeq x^{3\alpha_1 - 2\alpha_2} \frac{\xi_d^2 z_{33}^* (\mathbf{z}_3 \cdot \bar{\mathbf{m}}_1^*)}{\bar{m}_{11}^* |(\bar{\mathbf{z}}_1 \times \bar{\mathbf{m}}_1)_3|^2}. \end{aligned} \quad (25)$$

All off-diagonal elements of the V_{CKM} contain the F–N factors. This means that there is little difference between the diagonalization matrices for up-type quarks and for down-type quarks in $SU(2)_L$ -doublets. Further, V_{ub} is zero in the leading order but non-zero in the next-to-leading order, which implies that the element V_{ub} is naturally suppressed compared to V_{td} .

4 Charged lepton mass matrix

Effective Yukawa interactions among charged leptons are described as

$$W_E = \sum_{i,j=1}^3 \left[\mathcal{H}_{ij} H_{di} H_{uj} S_0 + \mathcal{M}_{ij} (L_i H_{uj} N_0^c + L_i E_j^c H_{d0}) \right], \quad (26)$$

where $\mathcal{H} = f_H \Gamma_2 H \Gamma_2$ with $\det H = 1$. The matrix H has no hierarchical structure. In the lepton sector, mixings occur between L and H_d which are $SU(2)_L$ -doublets. Consequently, the charged lepton mass matrix is expressed in terms of the 6×6 matrix [4, 5]

$$\widehat{\mathcal{M}}_l = \begin{matrix} & H_u^+ & E^{c+} \\ \begin{matrix} H_d^- \\ L^- \end{matrix} & \begin{pmatrix} \rho_S \mathcal{H} & 0 \\ \rho_N \mathcal{M} & \rho_d \mathcal{M} \end{pmatrix} \end{matrix} \quad (27)$$

in unit of M_S . The study of the charged lepton mass matrix is parallel to that of the down-type quark mass matrix in the previous section. The matrix $\widehat{\mathcal{M}}_l$ is diagonalized via bi-unitary transformation as

$$\widehat{\mathcal{V}}_l^{-1} \widehat{\mathcal{M}}_l \widehat{\mathcal{U}}_l = \text{diag}(\Lambda_l^{(0)}, \epsilon_d \Lambda_l^{(2)}). \quad (28)$$

Among six eigenvalues three of them represent heavy modes with the GUT scale masses. The remaining three, corresponding to charged leptons, are derived from the diagonalization

$$\epsilon_d^2 \mathcal{V}_l^{-1} (A_l^{-1} + B_l^{-1})^{-1} \mathcal{V}_l = (\epsilon_d \Lambda_l^{(2)})^2, \quad (29)$$

where $A_l = |\rho_S|^2 \mathcal{H}^\dagger \mathcal{H}$ and $B_l = |\rho_N|^2 \mathcal{M}^\dagger \mathcal{M}$. Charged lepton masses are given by $M_S \epsilon_d \Lambda_l^{(2)} = \text{diag}(m_e, m_\mu, m_\tau)$. Explicit forms are

$$(m_e, m_\mu, m_\tau) \simeq v_{d0} f_M x^{\alpha_1} \times \left(\frac{1}{\sqrt{a'}} x^{\beta_1}, \sqrt{\frac{a'}{b'}} x^{\beta_2}, \sqrt{\frac{b'}{c'}} \right), \quad (30)$$

where

$$\begin{aligned} a' &= \xi_l^2 |\overline{h'}_{11}|^2 + |\overline{m'}_{11}|^2, & b' &= \xi_l^2 |(\overline{h'}_1 \times \overline{m'}_1)_3|^2, \\ c' &= \xi_l^4 x^{2(\beta_1 - \beta_2)} |(\mathbf{h}'_3 \cdot \overline{\mathbf{m'}}_1)|^2 + \xi_l^2 x^{2(\alpha_1 - \alpha_2)} |(\mathbf{m}'_3 \cdot \overline{\mathbf{h'}}_1)|^2, \\ \xi_l^2 &= \left| \frac{\rho_N f_M}{\rho_S f_H} \right|^2 x^{2(\alpha_1 - \beta_1)} = \xi_d^2 \left| \frac{f_Z}{f_H} \right|^2 x^{4(\alpha_1 - \beta_1)}. \end{aligned} \quad (31)$$

Here we use the notations $\overline{m'}_{ij} = \Delta(M)_{ji} = \overline{m}_{ji}^*$, $h'_{ij} = (H^\dagger)_{ij}$, $\overline{h'}_{ij} = \Delta(H)_{ji}$ and

$$\begin{aligned} \mathbf{m}'_i &= (m_{i1}, m_{i2}, m_{i3})^\dagger, & \overline{\mathbf{m'}}_i &= (\overline{m}_{i1}, \overline{m}_{i2}, \overline{m}_{i3})^\dagger, \\ \mathbf{h}'_i &= (h'_{1i}, h'_{2i}, h'_{3i})^T, & \overline{\mathbf{h'}}_i &= (\overline{h'}_{1i}, \overline{h'}_{2i}, \overline{h'}_{3i})^T. \end{aligned}$$

The diagonalization matrix is of the form

$$\mathcal{V}_l = (\mathbf{w}_1^{(l)}, \mathbf{w}_2^{(l)}, \mathbf{w}_3^{(l)}), \quad (32)$$

$$\mathbf{w}_1^{(l)} = N_1^{(l)} \begin{pmatrix} 1 \\ u_1^{(l)} \\ v_1^{(l)} \end{pmatrix}, \quad \mathbf{w}_2^{(l)} = N_2^{(l)} \begin{pmatrix} u_2^{(l)} \\ 1 \\ v_2^{(l)} \end{pmatrix}, \quad \mathbf{w}_3^{(l)} = N_3^{(l)} \begin{pmatrix} u_3^{(l)} \\ v_3^{(l)} \\ 1 \end{pmatrix}.$$

Each element is given by

$$\begin{aligned} u_1^{(l)} &\simeq \sigma_1 x^{\beta_1 - \beta_2}, & v_1^{(l)} &\simeq \sigma_2 x^{\beta_1}, \\ u_2^{(l)} &\simeq -(u_1^{(l)})^*, & v_2^{(l)} &\simeq \sigma_4 x^{\beta_2}, \\ u_3^{(l)} &\simeq \sigma_3^* x^{\beta_1}, & v_3^{(l)} &\simeq -(v_2^{(l)})^* \end{aligned}$$

and

$$N_i^{(l)} = \left(1 + |u_i^{(l)}|^2 + |v_i^{(l)}|^2 \right)^{-1/2} \quad (i = 1, 2, 3),$$

where

$$\begin{aligned} \sigma_1 &= \frac{\xi_l^2 \bar{h}'_{21} \bar{h}'_{11}^* + \bar{m}'_{21} \bar{m}'_{11}^*}{\xi_l^2 |\bar{h}'_{11}|^2 + |\bar{m}'_{11}|^2}, & \sigma_2 &= \frac{\xi_l^2 \bar{h}'_{31} \bar{h}'_{11}^* + \bar{m}'_{31} \bar{m}'_{11}^*}{\xi_l^2 |\bar{h}'_{11}|^2 + |\bar{m}'_{11}|^2}, \\ \sigma_3 &= \frac{(\bar{\mathbf{h}}'_1 \times \bar{\mathbf{m}}'_1)_1}{(\bar{\mathbf{h}}'_1 \times \bar{\mathbf{m}}'_1)_3}, & \sigma_4 &= -\frac{(\bar{\mathbf{h}}'_1 \times \bar{\mathbf{m}}'_1)_2}{(\bar{\mathbf{h}}'_1 \times \bar{\mathbf{m}}'_1)_3}. \end{aligned} \tag{33}$$

5 Neutrino mass matrix

In the neutral lepton sector we have the 15×15 mass matrix [4, 5]

$$\widehat{\mathcal{M}}_{NS} = \begin{matrix} & \begin{matrix} H_u^0 & H_d^0 & L^0 & N^c & S \end{matrix} \\ \begin{matrix} H_u^0 \\ H_d^0 \\ L^0 \\ N^c \\ S \end{matrix} & \begin{pmatrix} 0 & \rho_S \mathcal{H} & \rho_N \mathcal{M}^T & 0 & \rho_d \mathcal{M}^T \\ \rho_S \mathcal{H} & 0 & 0 & 0 & \rho_u \mathcal{M}^T \\ \rho_N \mathcal{M} & 0 & 0 & \rho_u \mathcal{M} & 0 \\ 0 & 0 & \rho_u \mathcal{M}^T & \mathcal{N} & 0 \\ \rho_d \mathcal{M} & \rho_u \mathcal{M} & 0 & 0 & \mathcal{S} \end{pmatrix} \end{matrix}, \tag{34}$$

where $\rho_u = \langle H_{u0} \rangle / M_S = v_{u0} / M_S$ and \mathcal{N} , \mathcal{S} stand for Majorana mass matrices for the superfields N^c and S with odd R-parity. This mass matrix comes from the effective

Yukawa interactions

$$\begin{aligned}
W_{NS} = & \sum_{i,j=1}^3 \mathcal{H}_{ij} H_{di} H_{uj} S_0 + \sum_{i,j=1}^3 \mathcal{M}_{ij} (L_i H_{uj} N_0^c + L_i N_j^c H_{u0}) \\
& + \sum_{i,j=1}^3 \mathcal{M}_{ij} (S_i H_{uj} H_{d0} + S_i H_{dj} H_{u0})
\end{aligned} \tag{35}$$

and from Majorana mass terms for N^c and S . Here we assume (the electroweak scale) \ll (\mathcal{N} scale) \ll (\mathcal{S} scale). The matrix \mathcal{N} has the form

$$\mathcal{N} = f_N \Gamma_2 N \Gamma_2,$$

in which N contains no hierarchical structure and $\det N = 1$.

Mixings in the lepton sector occur between $SU(2)_L$ -doublet fields L and H_d . When we diagonalize the charged lepton mass matrix, the neutral leptons in $SU(2)_L$ -doublet undergo the same transformation as the diagonalization matrix for charged leptons. In addition, the seesaw mechanism is at work. Hence, neutrino mass matrix for light modes becomes

$$\mathcal{M}_\nu = M_S \epsilon_u^2 \Lambda_l^{(2)} \mathcal{V}_l^\dagger \mathcal{N}^{-1} \mathcal{V}_l^* \Lambda_l^{(2)} \tag{36}$$

in the diagonal mass basis for charged leptons, where $\epsilon_u = \rho_u / \rho_N = v_{u0} / \langle N_0^c \rangle = \mathcal{O}(10^{-15})$. By diagonalizing \mathcal{M}_ν we obtain neutrino masses

$$\mathcal{V}_\nu^T \mathcal{M}_\nu \mathcal{V}_\nu = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) \tag{37}$$

and the MNS matrix [7]

$$V_{\text{MNS}} = \mathcal{V}_\nu^T. \tag{38}$$

The matrix \mathcal{M}_ν is rewritten as

$$\mathcal{M}_\nu = \frac{M_S \epsilon_u^2}{f_N} Y^T N^{-1} Y, \tag{39}$$

where

$$Y = \Gamma_2^{-1} \mathcal{V}_l^* \Lambda_l^{(2)}. \tag{40}$$

As seen in Eq.(32), \mathcal{V}_l is of the form

$$\mathcal{V}_l \simeq \begin{pmatrix} 1 & -\sigma_1^* x^{\beta_1 - \beta_2} & \sigma_3^* x^{\beta_1} \\ \sigma_1 x^{\beta_1 - \beta_2} & 1 & -\sigma_4^* x^{\beta_2} \\ \sigma_2 x^{\beta_1} & \sigma_4 x^{\beta_2} & 1 \end{pmatrix}. \tag{41}$$

In addition, from Eq.(30) the matrix $\Lambda_l^{(2)}$ is described as

$$\Lambda_l^{(2)} \propto \Gamma_2 \times \text{diag} \left(\frac{1}{\sqrt{a'}}, \sqrt{\frac{a'}{b'}}, \sqrt{\frac{b'}{c'}} \right). \quad (42)$$

Hence, $Y = (\mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3)$ is given by

$$\begin{aligned} \mathbf{y}_1 &\simeq \rho_N f_M x^{\alpha_1} \frac{1}{\sqrt{a'}} \begin{pmatrix} 1 \\ \sigma_1^* x^{2(\beta_1 - \beta_2)} \\ \sigma_2^* x^{2\beta_1} \end{pmatrix}, \\ \mathbf{y}_2 &\simeq \rho_N f_M x^{\alpha_1} \sqrt{\frac{a'}{b'}} \begin{pmatrix} -\sigma_1 \\ 1 \\ \sigma_4^* x^{2\beta_2} \end{pmatrix}, \\ \mathbf{y}_3 &\simeq \rho_N f_M x^{\alpha_1} \sqrt{\frac{b'}{c'}} \begin{pmatrix} \sigma_3 \\ -\sigma_4 \\ 1 \end{pmatrix}, \end{aligned} \quad (43)$$

where σ_i 's ($i = 1 \sim 4$) are $\mathcal{O}(1)$. We note that in the upper triangular elements of Y the F–N factors x^{β_1} and x^{β_2} cancel out. Introducing the notation

$$\Lambda_\kappa = \text{diag} \left(\sqrt{\frac{c'}{a' b'}}, \frac{\sqrt{c' a'}}{b'}, 1 \right) = \text{diag} (\kappa_1, \kappa_2, 1), \quad (44)$$

we have

$$\mathcal{M}_\nu = \frac{v_{u0}^2 f_M^2}{M_N} x^{2\alpha_1} \frac{b'}{c'} \times \Lambda_\kappa \Sigma \Lambda_\kappa \quad (45)$$

with $M_N = f_N M_S$, which represents the typical scale of the R-handed neutrino Majorana masses. In the present model the F–N factors $x^{2\beta_1}$, $x^{2\beta_2}$ and $x^{2(\beta_1 - \beta_2)}$ are sufficiently small compared to 1. Therefore, Σ is approximated as

$$\Sigma \simeq \begin{pmatrix} 1 & 0 & 0 \\ -\sigma_1 & 1 & 0 \\ \sigma_3 & -\sigma_4 & 1 \end{pmatrix} \times N^{-1} \times \begin{pmatrix} 1 & -\sigma_1 & \sigma_3 \\ 0 & 1 & -\sigma_4 \\ 0 & 0 & 1 \end{pmatrix}. \quad (46)$$

6 Numerical analysis

In a simple setting $N = \mathbf{1}$ and $\Lambda_\kappa = \mathbf{1}$, the matrix V_{MNS} is nothing but diagonalization matrix of Σ . Further, when we put $\sigma_1 = \sigma_4 = 2.2$ and $\sigma_3 = 1.2$ in Eq.(46), the eigenvalues of Σ turn out to be 0.039, 2.122 and 11.959. It follows that the neutrino mass ratio is

$$\left(\frac{\Delta m_{32}^2}{\Delta m_{21}^2} \right)^{1/2} = 5.57 \quad (47)$$

for the normal hierarchy, which is consistent with experimental data: 5.56 ± 0.20 . The observed absolute value of $\Delta m_{32}^2 \simeq m_{\nu_3}^2$ is obtained by taking $M_N = \mathcal{O}(10^9)\text{GeV}$. The MNS matrix is estimated as

$$V_{\text{MNS}} = \begin{pmatrix} 0.865 & 0.471 & 0.172 \\ -0.455 & 0.594 & 0.663 \\ 0.211 & -0.652 & 0.728 \end{pmatrix}. \quad (48)$$

Experimental data provided that $\delta_{CP} = 0$ show [10]

$$V_{\text{MNS}} = \begin{pmatrix} 0.824 & 0.547 & 0.145 \\ -0.499 & 0.583 & 0.641 \\ 0.266 & -0.601 & 0.754 \end{pmatrix}. \quad (49)$$

Although our setting $N = \mathbf{1}$ and $\Lambda_\kappa = \mathbf{1}$ is very simple, all elements offered here are consistent with the observed values within 20% including $|(V_{\text{MNS}})_{13}| = |\sin \theta_{13}|$.

Finally we discuss fermion mass spectra and the CKM matrix. The present setting $\Lambda_\kappa = \mathbf{1}$ ($\kappa_1 = \kappa_2 = 1$) means

$$m_e : m_\mu : m_\tau = x^{\beta_1} : x^{\beta_2} : 1. \quad (50)$$

From experimental values of masses we have $x^{\beta_1} = \lambda^{5.6}$ and $x^{\beta_2} = \lambda^{1.9}$ with $\lambda = 0.23$. As to the other parameters, we set $x^{\alpha_1} = \lambda^{3.3}$, $x^{\alpha_2} = \lambda^{2.3}$ for the F-N factor Γ_1 and $\xi_d = \lambda^{-0.2}$, $\xi_l = \lambda^{-5}$ which leads to

$$f_Z = \frac{\rho_N}{\rho_S} f_M \lambda^{2.5}, \quad f_H = \frac{\rho_N}{\rho_S} f_M \lambda^{2.7}, \quad (51)$$

respectively. In this setting of parameter values, each first term in Eq.(21) for a and c , and Eq.(31) for a' and c' is dominant. From Eqs.(13), (20) and (25), we have

$$\frac{m_s}{m_u} V_{us} \simeq \frac{v_{d0}}{v_{u0}}, \quad (52)$$

Table 1: Quark and lepton masses divided by top quark mass ($\lambda = 0.23$)

mass ratio	our result	observed values
m_c/m_t	$\lambda^{3.3}$	$\lambda^{3.34}$
m_u/m_t	$\lambda^{7.7}$	$\lambda^{7.65}$
m_b/m_t	$\lambda^{2.5}$	$\lambda^{2.54}$
m_s/m_t	$\lambda^{5.1}$	$\lambda^{5.11}$
m_d/m_t	$\lambda^{7.1}$	$\lambda^{7.14}$
m_τ/m_t	$\lambda^{3.1}$	$\lambda^{3.12}$
m_μ/m_t	$\lambda^{5.0}$	$\lambda^{5.04}$
m_e/m_t	$\lambda^{8.7}$	$\lambda^{8.67}$

which is independent of values of α_i and β_i ($i = 1, 2$). The observed value of the left hand side leads to $(v_{d0}/v_{u0}) \simeq 10.5 = \lambda^{-1.6}$, which is reverse to the usual solution with large $\tan \beta = v_u/v_d$. This fact suggests that Higgs fields other than H_{u0} and H_{d0} develop their VEV's. It is expected that there exist rich spectra of Higgs fields beyond those of the MSSM at the TeV scale. We take $(v_{d0}/v_{u0}) = \lambda^{-1.6}$ as an input of the present analysis. In the present setting most of fermion mass hierarchies are attributed to the F–N factors. In order to reproduce observed mass spectra and the CKM matrix precisely, we adjust the other parameters as

$$\begin{aligned}
m_{33} &= \lambda^{0.7}, & z_{33} &= \lambda^{-0.3}, \\
\overline{m}_{11} &= \lambda^{0.5}, & \overline{z}_{11} &= \lambda^{-0.3}, & \overline{h'}_{11} &= \lambda^{2.9}, \\
|(\overline{z}_1 \times \overline{m}_1)_3| &= \lambda^{0.2}, & |(\mathbf{z}_3^* \cdot \overline{m}_1)| &= \lambda^{0.2}, & |(\mathbf{m}_3 \cdot \overline{z}_1^*)| &= \lambda^{0.3}, \\
|(\overline{h'}_1 \times \overline{m'}_1)_3| &= \lambda^{0.8}, & |(\mathbf{h}_3'^* \cdot \overline{m'}_1)| &= \lambda^0.
\end{aligned} \tag{53}$$

As seen in Tables 1 and 2, our results are in good agreement with observed values. Although there are many parameters, it is noteworthy that these parameters other than $\overline{h'}_{11}$ are $\mathcal{O}(1)$. Note that this parameter $\overline{h'}_{11}$ stands for the cofactor $\Delta(H)_{11}$.

We may change the above values of the parameters α_i and β_i ($i = 1, 2$). As another setting of the F–N factors Γ_1 and Γ_2 , we put

$$x^{\alpha_1} = \lambda^{4.6}, \quad x^{\alpha_2} = \lambda^{2.6}, \quad x^{\beta_1} = \lambda^{5.6}, \quad x^{\beta_2} = \lambda^{1.9}. \tag{54}$$

In this case the relation

$$m_d : m_s : m_b = x^{\alpha_1} : x^{\alpha_2} : 1 \tag{55}$$

Table 2: Elements of the CKM matrix

$(V_{\text{CKM}})_{ij}$	our result	observed values
V_{us}	$\lambda^{1.0}$	$\lambda^{1.02}$
V_{cb}	$\lambda^{2.2}$	$\lambda^{2.17}$
V_{td}	$\lambda^{3.2}$	$\lambda^{3.23}$
V_{ub}	$\lambda^{3.9}$	$\lambda^{3.87}$

holds together with Eq.(50). Further, we take

$$f_Z = \frac{\rho_N}{\rho_S} f_M \lambda^{1.6}, \quad f_H = \frac{\rho_N}{\rho_S} f_M \lambda^{2.5}, \quad (56)$$

corresponding to $\xi_d = \lambda^{-0.6}$, $\xi_l = \lambda^{-3.5}$ and

$$\begin{aligned}
m_{33} &= \lambda^{1.23}, & z_{33} &= \lambda^{0.03}, \\
\overline{m}_{11} &= \lambda^{1.27}, & \overline{z}_{11} &= \lambda^{0.87}, & \overline{h'}_{11} &= \lambda^{2.17}, \\
|(\overline{\mathbf{z}}_1 \times \overline{\mathbf{m}}_1)_3| &= \lambda^{1.13}, & |(\mathbf{z}_3^* \cdot \overline{\mathbf{m}}_1)| &= \lambda^0, & |(\mathbf{m}_3 \cdot \overline{\mathbf{z}}_1^*)| &= \lambda^{0.7}, \\
|(\overline{\mathbf{h}'}_1 \times \overline{\mathbf{m}'}_1)_3| &= \lambda^{0.83}, & |(\mathbf{h}'_3^* \cdot \overline{\mathbf{m}'}_1)| &= \lambda^{-0.7}.
\end{aligned} \quad (57)$$

In this choice of the parameter values our results remain the same as given in Tables 1 and 2.

7 Summary

We have studied flavor mixings, especially the MNS matrix, in detail in the $SU(6) \times SU(2)_R$ model, in which the F–N mechanism plays an important role. Due to the F–N mechanism both effective Yukawa couplings and R-handed neutrino Majorana mass matrix have hierarchical structure, which is described in terms of the F–N factors. In this model the D^c – g^c and L – H_d mixings as well as generation mixings occur and affect both fermion mass spectra and flavor mixings.

In the D^c – g^c mixings, since D^c and g^c are both $SU(2)_L$ -singlets, the disparity between the diagonalization matrices for up-type quarks and down-type quarks in $SU(2)_L$ -doublets is rather small. Accordingly, V_{CKM} exhibits small mixing. On the other hand, in the L – H_d mixings, since L and H_d are both $SU(2)_L$ -doublets, there appears no disparity between the diagonalization matrices for charged leptons and neutrinos unless the seesaw mechanism does not take place. As a matter of fact, however, the seesaw mechanism is at work

and an additional transformation is required to diagonalize the neutrino mass matrix. This additional transformation matrix yields nontrivial V_{MNS} . In the present model the neutrino mass matrix has characteristic structure as seen in Eqs.(45) and (46). This is due to the fact that the F–N factors cancel out in the upper triangular elements of Y . As a consequence, there is no hierarchical structure in \mathcal{M}_ν and eventually V_{MNS} exhibits large mixing.

The characteristic structure of fermion spectra is attributed to the hierarchical effective Yukawa couplings due to the F–N mechanism and also to the D^c – g^c and L – H_d mixings. In particular, the difference of mass hierarchies among up-type quarks, down-type quarks and charged leptons has its origin in D^c – g^c and L – H_d mixings. In the neutrino sector we have to incorporate the Majorana mass hierarchy of R-handed neutrinos with the seesaw mechanism. Numerical results are consistent with all observed values of fermion masses and mixings. The present model provides a unified description of mass spectra and flavor mixings.

In order to determine the F–N factors and the magnitude of f_i 's ($i = M, Z, H, N$) theoretically, we need an appropriate flavor symmetry and also the flavor charge assignment to matter fields. In the previous works we made an attempt to find several solutions [14, 15]. The detailed study of this issue is the subject of future works. Further, in this paper we ignored the phase factors of VEV's for matter fields and assumed $\delta_{CP} = 0$. The study of the CP-violation will be carried out elsewhere.

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